[A large fraction of the contents are adapted from Prof. Kunz's Lecture Notes on Iroduction to Plasma Astrophysics, (I) Part I]

# Intro to Plasma Physics.

- What's a plasma?
- collective behaviours?
- difference from gas?
- characteristic scales?

1). Ionized gas.

- Saha eqn., quantifying ionization.
  derived under quantum & statistical mechanis

for a gas in thermal equilibrium:

$$\frac{n_i}{n_n} \simeq 2.4\times10^{21}\ \frac{T^{3/2}}{n_i}\left(e^{-\varepsilon_i/k_BT}\right).$$

$n_i$: # density of ionized atoms

$n_n$: - - - - - - - neutral atoms.

$\varepsilon_i$: ionization energy of gas (to remove the outmost $e^-$ from an atom)

When $\varepsilon_i \sim$ few times $k_BT$, ionization increases abruptly.

$k_B$: Boltzmann constant.

$k_B \simeq 1.38\times10^{-23}$ J/K.

$\simeq 8.62\times10^{-5}$ eV/K.

$1\,\text{eV} \sim 1.6\times10^{-19}\,\text{J} \Leftrightarrow 1.16\times10^4\,\text{K}.$

In the rest of the notes, we write $T$ for $k_BT$,

$T$ will have the unit (~~dim~~ of eV.

2) Are quantum effects important in plasmas?

Let's compare $\lambda$ and $\Delta r$.

$\lambda$: de Broglie wavelength (of matter wave)

$\Delta r$: inter-particle distance.

if $\lambda \ll \Delta r$, particles are distinguishable, quantum effects not important.

$\lambda = \frac{h}{p}$     $h$: Planck constant

$hc \simeq 1\,\mathrm{keV \cdot nm}$

$p \sim \sqrt{mT}$,    $T \sim 10\,\mathrm{keV}$ for tokamak plasma

$m_e c^2 \sim 500\,\mathrm{keV}$

$$\lambda \simeq \frac{hc}{\sqrt{mc^2 T}} \sim 10^{-11}\,\mathrm{m}$$

$\Delta r \sim n^{-1/3}$    $n \sim 10^{21}\,\mathrm{m^{-3}}$ for tokamak plasma

$\Delta r \sim 10^{-7}\,\mathrm{m}$.

So: $\lambda \ll \Delta r$, classic description is enough for many plasmas

3) If plasmas are close to thermodynamical equilibrium, Boltzmann statistics apply.

plasmas follow Maxwell-Boltzmann distribution.

$$f_M(v) = \frac{n}{\pi^{3/2} v_{th}^3} \exp\left[-\frac{v^2}{v_{th}^2}\right]$$

$v_{th} \equiv \sqrt{\frac{2T}{m}}$

3) Collective behaviours

Kinetic energy of the plasma $\gg$ potential energy due to Coulomb interaction between binary pairs.

$$\boxed{T \gg e\phi}$$

$\phi \sim \frac{e}{\delta r} \sim e n^{1/3}$

↳ Coulumb potential.

we will illustrate this point later.

4) Quasi-neutrality & Debye Shielding

a plasma, generally speaking, is a good conductor. it shields external $E$ fields.

Let's consider its <u>dielectric</u> phenomena

consider a plasma in local thermodynamical equilibrium

species $j'$, charge $q_{j'}$

Boltzmann distribution for density

$$n_{j'} = \bar{n}_{j'} \exp\left[-\frac{q_{j'}\phi(\vec{r})}{T}\right]$$

$$\simeq \bar{n}_{j'}\left[1 - \frac{q_{j'}\phi(\vec{r})}{T}\right]$$

← $T \gg q_{j'}\phi$ Taylor expansion.

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a "fluid" way to derive $n_{j'}$.

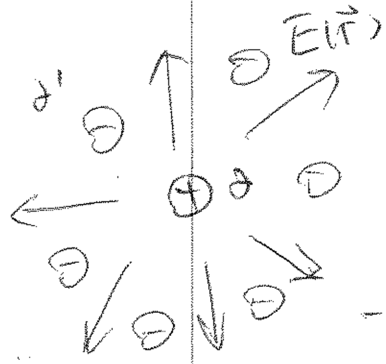
$$\rho \frac{d\vec{v}_{j'}}{dt} = n_{j'} q_{j'} \vec{E} - \vec{\nabla} P = 0.$$

(steady state)

$\vec{E} = -\nabla\phi \qquad P = n_j T$

$n_{j'} q_{j'} \nabla\phi = T \nabla n_{j'} \qquad \nabla \to \frac{\partial}{\partial r}$

$$n_{j'} = \bar{n}_{j'} \exp\left(-\frac{q_{j'}\phi}{T}\right)$$




What's "$\phi$"?

$$-\nabla^2\phi = \nabla\cdot\vec{E} = 4\pi q_\sigma \delta(\vec{r}) + 4\pi \sum_{\sigma'} q_{\sigma'} n_{\sigma'}$$

$$\simeq 4\pi q_\sigma \delta(\vec{r}) + \underbrace{4\pi \sum_{\sigma'} q_{\sigma'} \bar{n}_{\sigma'}}_{\to 0 \text{ charge-neutral overall}} - \underbrace{\left[\sum_{\sigma'} \frac{4\pi q_{\sigma'}^2 \bar{n}_{\sigma'}}{T}\right]}_{\text{define as } \lambda_{De}^{-2}} \phi$$

$\lambda_{De}$: Debye length.

under spherical symmetry:

$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial \phi}{\partial r} - \frac{1}{\lambda_D^2}\phi = -4\pi q_\sigma \delta(\vec{r})$$

solution $\boxed{\phi = \frac{q_\sigma}{r}\exp\left(-\frac{r}{\lambda_D}\right)}$ $\quad \phi \to \frac{q_\sigma}{r}$ as $r \to 0$

$\phi \to 0$ as $r \to \infty$

the potential of a single charge is exponentially shielded on the length scale $\lambda_{De}$

→ Debye shielding

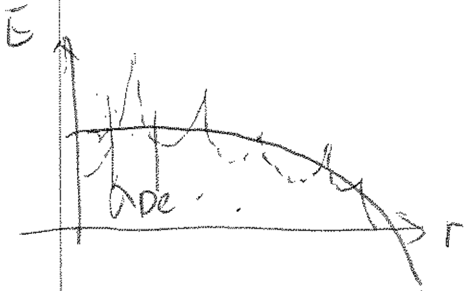
→ Debye sphere / cloud.

- dielectric behaviour, shield $\vec{E}$ by polarization.
- the Coulumb potential of every single charge is Debye shielded by other charges.
- at scale $l \gg \lambda_D$: $n_i \simeq n_e$ ($\simeq n$, "plasma density")

$$\Downarrow$$

<u>Quasi-neutrality</u>

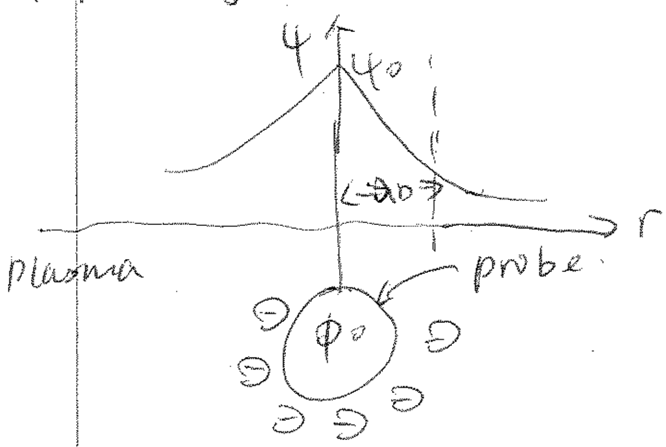
- collective behavior/effects $\gg$ binary collisions (distinguish from a gas)




• $\vec{E}$ fields are coarse-grained at scale $\sim\lambda_D$,

create macroscopic E fields smoothly vary in time & space

if putting a conductor (probe) into a plasma.




plasma will form a "sheath", w/ the thickness $\lambda_{De} = \sqrt{\frac{T}{4\pi n e^2}}$.

## Plasma Parameter

$\Lambda \equiv \lambda_D^3 n_e$, $\Lambda = \lambda_D^2 n_e \lambda_D = \frac{T}{4\pi n e^2} n \lambda_D$

$$= \frac{T}{4\pi n e^2 / \lambda_D} \simeq \frac{\text{Kinetic } E}{\text{potential } E}.$$

Dybey shielding meaningful only if there are enough particles in a Debye sphere: $\Lambda \gg 1$.

$\Lambda \gg 1$: • $n$ distribution uniform at $l \gg \lambda_D$.

plasma is quasi-neutral.

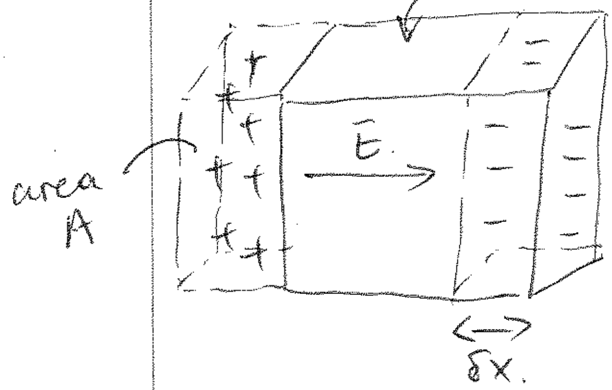
• weakly coupled

collective electrostatic interations $\gg$ binary particle collisions

5) plasma Oscillation.

Instead of DC Efield, consider a time-dependent $E$, or a perturbation.

spatially uniform, quasi-neutral plasma, equilibrated Debye sphere.




shift one species by $\delta x$

$$\oint \vec{E} \cdot d\vec{S} = 4\pi \int \rho \, dV$$

$$EA = 4\pi e n_e A \delta x$$

$$E = 4\pi e n_e \delta x$$

Equation of motion for $e^-$.

$$m_e \frac{d^2 \delta x}{dt^2} = -eE = -4\pi e^2 n_e \delta x$$

Define $\omega_{pe}^2 = \frac{4\pi e^2 n_e}{m_e}$,

$$m_e \frac{d^2 \delta x}{dt^2} + m_e \omega_{pe}^2 \delta x = 0.$$

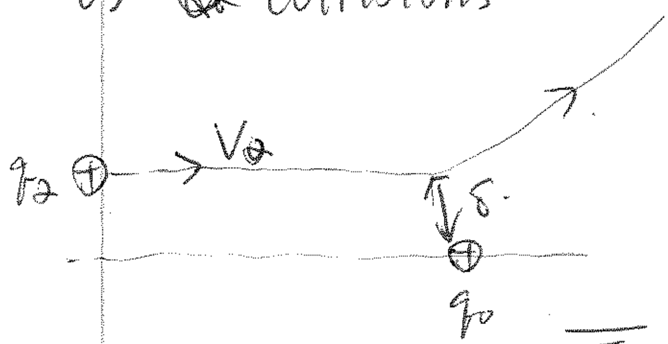
→ Eqn for harmonic oscillator w/ frequency $\omega_{pe}$.

→ Langmuir oscillation.

→ collective response of plasma to disturbance to go back to quasi-neutrality

$$\boxed{\omega_{pe} \sim \frac{v_{the}}{\lambda_{De}}}$$

6) Collisions.



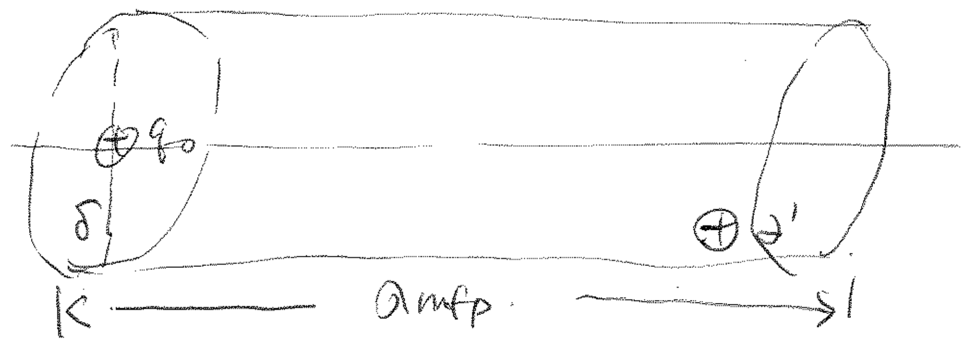

$$\frac{1}{2} m v_2^2 \simeq \frac{q_2 q_0}{\delta}$$

$$\delta \sim \frac{2 q_0 q_2}{m v_2^2}$$

(Landau length)

$$v_2 \sim v_{th} \sim \sqrt{\frac{2T}{m_2}}$$

$\sigma \sim \delta^2$ roughly gives the cross section of collision.




$\lambda_{mfp} \sim v_2 \tau$. $\tau$. collision time.

$$\sigma \cdot \lambda_{mfp} \cdot n \sim 1.$$

$$\lambda_{mfp} \sim \frac{1}{n\sigma} \sim \frac{T^2}{ne^4},$$ hotter plasma, longer $\lambda_{mfp}$.

for neutral gas: $\lambda_{mfp} \sim \frac{1}{n\sigma} \sim \frac{1}{n}$, $T$ independent.

$$\frac{\lambda_{mfp}}{\lambda_D} \sim \frac{T^2}{ne^4}\left(\frac{ne^2}{T}\right)^{1/2} \sim n\lambda_D^3 \sim \Lambda \gg 1.$$

→ macroscopic field, collective dynamics

≫ microscopic field, individual collisions.

collision frequency, $\nu \sim \frac{1}{\tau} \sim n\sigma V_{th} \sim \frac{ne^4}{m^2 T^{3/2}}$

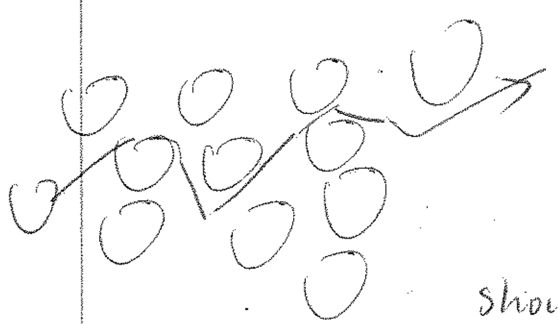
$$\frac{\omega_{pe}}{\nu} \sim \left(\frac{ne^2}{m}\right)^{1/2}\left(\frac{m^2 T^{3/2}}{ne^4}\right) \sim n\lambda_D^3 \sim \Lambda \gg 1.$$

$\tau \sim 1/\nu \quad \gg \quad 1/\omega_{pe}$.

time scale for collisional relaxation $f(v) \to f_M(v)$

time scale to establish debye shielding & Quasi-neutrality.

So: collisions occur between Debye spheres, instead of individual particles.



~~small-s~~

small-angle scattering dominates.,

should include a "$\ln\Lambda$" correction to collision frequency.

7). Define a plasma.

- collective behaviours dominate, low collisionality.
  $\Lambda \equiv n\lambda_D^3 \gg 1$.
- $\lambda_D \ll L$. shield DC $\vec{E}$ field.
- $\omega_p \gg \omega$. oscillationary response to time-dependent field.

Other parameters / scales.

Larmor radius., freq. $\rho_\alpha \equiv \dfrac{v_{th\alpha}}{\Omega_\alpha}$. $\Omega_\alpha \equiv \dfrac{q_\alpha B}{m_\alpha c}$.

plasma inertial length. $d_\alpha \equiv \dfrac{c}{\omega_{p\alpha}}$ $= \left(\dfrac{m_\alpha c^2}{4\pi n_\alpha e^2}\right)^{1/2}$ , $\omega_{p\alpha} = \sqrt{\dfrac{4\pi n_\alpha e^2}{m_\alpha}}$

plasma $\beta$.

$$\beta_\alpha \equiv \frac{n_\alpha T_\alpha}{B^2/8\pi} = \frac{\text{plasma pressure}}{\text{magnetic pressure}}$$

$$= \frac{2T_\alpha}{m_\alpha} \cdot \frac{4\pi m_\alpha n_\alpha}{B^2}$$

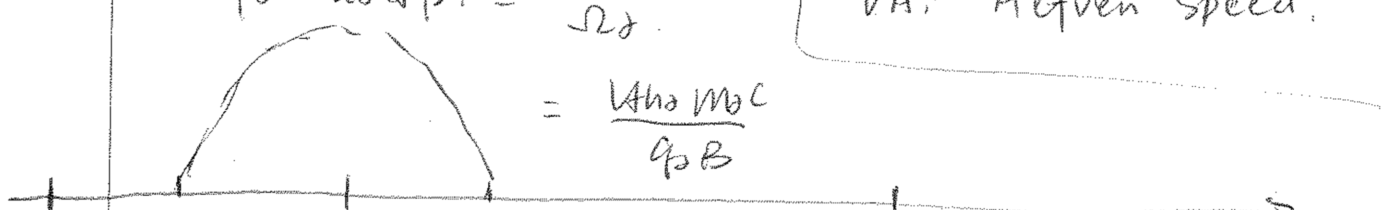
$$= \frac{v_{th\alpha}^2}{V_{A\alpha}^2} = \left(\frac{\rho_\alpha}{d_\alpha}\right)^2$$

$V_A$: Alfvén speed.

Scale comparison.

$\rho_\alpha = d_\alpha \sqrt{\beta} = \dfrac{v_{th\alpha}}{\Omega_\alpha}$

$= \dfrac{v_{th\alpha} m_\alpha c}{q_\alpha B}$




$\lambda_{De} = \dfrac{v_{th}}{\omega_{pe}} = \sqrt{\dfrac{T}{4\pi n e^2}}$

$d_\alpha = \dfrac{c}{\omega_{pe}} = \sqrt{\dfrac{m_\alpha c^2}{4\pi n_\alpha e^2}}$